Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$-v \sec \phi \text{ along } EP,$$

and

$$-v \tan \phi \text{ to the north.}$$

Its resulting acceleration to the east is therefore

$$2\,\omega\, v(\sec \phi - \tan \phi \sin \phi).$$

That is,

$$2\,\omega\, v \cos \phi \text{ to the east.} \tag{7}$$

The total displacement in a fall from a height h is therefore given approximately by writing $v=gt$, integrating with respect to t twice and substituting $t=(2\,h/g)^{\frac{1}{2}}$. The displacement is found to be

$$\tfrac{1}{3}\,\omega \cos \phi(8\,h^3/g^3)^{\frac{1}{2}}. \tag{8}$$

Finally, there is one component of relative acceleration which has not been written down; that is the vertical component due to the velocity v from west to east. On reference to equation (3), it is seen that this component is

$$2\,\omega\, v \cos \phi \text{ vertically upwards.} \tag{9}$$

7. *The general equations of relative motion.* If we take axes x, y, and z to be east, north, and vertically upwards, respectively, the preceding results are expressed by the usual equations for the motion of a body moving freely:

$$\begin{aligned} x'' &= 2\,y'\,\omega \sin \phi - 2\,z'\,\omega \cos \phi, && \text{by (2), (7).}\\ y'' &= -2\,x'\,\omega \sin \phi, && \text{by (4).}\\ z'' &= 2\,x'\,\omega \cos \phi, && \text{by (9).} \end{aligned}$$

ON THE REPRESENTATION OF NUMBERS AS THE SUM OF TWO SQUARES.

By M. KABA in Collaboration with L. E. DICKSON.

Consider the representations expressible as the sum of the squares of two numbers. Following Jacobi's notation for the theta-function with a special argument, we have

$$\theta(k) = \sqrt{\frac{2\,k}{\pi}} = 1 + 2q + 2q^4 + 2q^9 + \ldots$$

And on the other hand,

$$\frac{2k}{\pi}=1+4\left[\frac{q}{1-q}-\frac{q^3}{1-q^3}+\dots\right]$$

Comparing these two results we have

$$(1)\qquad 1+4\left[\frac{q}{1-q}-\frac{q^3}{1-q^2}+\frac{q^5}{1-q^5}-\dots\right]=(1+2q+2q^4+2q^9+\dots)^2.$$

When we develop the terms of the left member, we get

$$\begin{aligned}1+4[(q+q^2+q^3+\dots)-(q^3+q^6+q^9+\dots)\\+(q^5+q^{10}+q^{15}+\dots)-\dots]=1+4\Sigma N_e q^e.\end{aligned}$$

We note that the sign preceding a series is + or −, according as the first exponent is of the form $4m+1$ or $4m+3$. For

$$e=2^a p_1^{\pi_1} p_2^{\pi_2}\dots p_n^{\pi_n}\quad (p_1,\ \dots,\ p_n \text{ distinct odd primes}),$$

q^e occurs in those series, and only those, whose first exponents are of the form $p_1^{r_1}\dots p_n^{r_n}$ $(0\leq r_i\leq \pi_i)$. Let $e_i=+1$ or -1, according as p_i is of the form $4m+1$ or $4m+3$. But the product of two numbers each of the form $4m+1$, or each of the form $4m+3$, is of the form $4k+1$; while the product of $4m+1$ by $4l+3$ is of the form $4k+3$. Hence $p_1^{r_1}\dots p_n^{r_n}$ is of the form $4m+1$ or $4m+3$ according as $e_1^{r_1}\dots e_n^{r_n}=+1$ or -1. Thus

$$N_e=\Sigma\, e_1^{r_1}e_2^{r_2}\dots e_n^{r_n}\quad (0\leq r_i\leq \pi_i,\ i=1,\ \dots,\ n),$$

$$(2)\qquad N_e=(1+e_1+e_1^2+\dots+e_1^{\pi_1})(1+e_2+\dots+e_2^{\pi_2})\dots(1+e_n+\dots+e_n^{\pi_n}).$$

But in view of the right member of (1), N_e is also the number of ways of representing e as the sum of the squares of two integers, zero or positive, provided we regard j^2+k^2 and k^2+j^2 as distinct representations when $j\neq k$, $j\neq 0$, $k\neq 0$ (whereas $0+s^2$ and s^2+0 give the same representation). *The number* N_e *of such representations of* e *is given by* (2).

In case e has a prime factor of the form $4m+3$, then $e_i=-1$, and $1+e_i+\dots+e_i^{\pi_i}=0$ or 1, according as π_i is odd or even. Thus if π_i is odd there is no representation of e as the sum of two squares. If π_i is even, $N_e=N_{e'}$, where $e'=e/p_i^{\pi_i}$. If $N_e=0$, there is again no representation of e. For $N_{e'}>0$, there are two representations $e'=x'^2+y'^2$ and hence representations $e=x^2+y^2$, $x=x'p_i^{\pi_i/2}$, $y=y'p_i^{\pi_i/2}$. Since $N_e=N_e$, every representation of e may be derived from those of e' by multiplying the variables by $p_i^{\pi_i/2}$.

When the variables have a common factor >1, the representation is called improper.

Theorem. *There exists no representations as the sum of two squares for a number e having a prime factor of the form $4m+3$ occurring to an odd power; no proper representations when such a factor occurs to an even power. If $P=p_1^{\pi_1}\ldots p_s^{\pi_s}$, where $p_1, \ldots, p_s$ denote all the distinct primes of the form $4m+3$ which divide e, and if $\pi_1, \ldots, \pi_s$ are all even, there are as many improper representations of e as there are representations of e/P; every representation of e is of the type $(P^{\frac{1}{2}} x)^2+(P^{\frac{1}{2}} y)^2$.*

Examples. $N_e=0$ for $e=3, 7, 15, 21, 27, 63$; $N_9=1$, $N_{45}=2$, $N_{225}=3$, $45=3^2+6^2=6^2+3^2$, $9.25=15^2=9^2+12^2=12^2+9^2$.

The problem thus reduces to the case in which every prime factor of e is of the form $4m+1$. Then by (2),

$$N_e=(\pi_1+1)(\pi_2+1)\ldots(\pi_n+1). \tag{3}$$

For example, $N_5=N_{10}=2$, $N_{25}=3$, $25=5^2=3^2+4^2=4^2+3^2$.

When e is of the form $2^a p$, we have $N_e=2$. Removing the restriction that j^2+k^2 and k^2+j^2 shall be regarded as distinct representations, we obtain the well-known theorem of Fermat:

Every prime number p of the form $4m+1$ (and every product $2^a p$) can be represented in one and only one way as the sum of two squares.

In general, the representations enumerated in (3) include improper ones. For instance, if $\pi_1>1$, the $(\pi_1+1)(\pi_2+1)\ldots$ representations x^2+y^2 of e/p_1^2 yield improper representations $(p_1x)^2+(p_1y)^2$ of e. There are only 2^{n-1} distinct representations of e not distinguishing the order or signs of the variables in x^2+y^2 (Dirichlet-Dedekind, *Zahlentheorie*, p. 164).

A METHOD FOR CHANGING THE SCALE OF A NUMBER.

By C. E. WHITE, Nashville, Tennessee.

To convert a number in the scale s to a number in the scale $s-a$, where a may be positive or negative, the following process may be used.

Let $N=p_0p_1p_2p_3\ldots p_n=p_n+sp_{n-1}+s^2p_{n-2}+\ldots=f(s)$. By Taylor's theorem,

$$f(s)=f(a)+(s-a)f'(a)+\frac{(s-a)^2}{2!}f''(a)+\frac{(s-a)^3}{3!}f'''(a)+\ldots$$

Dividing $f(a)$ by $(s-a)$ we get $q_0+\frac{r_0}{s-a}$ where q_0 is the quotient and r_0 the remainder, or $f(a)=r_0+q_0(s-a)$.